Survey Team 2

A Survey of Recent Research on Early Childhood Mathematics Education

Iliada Elia[1], Anna E. Baccaglini-Frank[2], Esther Levenson[3], Nanae Matsuo[4], and Nosisi Feza[5]

ABSTRACT This is a summary report of the ICME-14 survey on Early Childhood Mathematics Education, which was conducted by the authors of this paper, with the aim to establish a review of the state-of-the-art of the most important developments, and of current tendencies, new perspectives and emerging challenges in the particular field. The survey was based on an analysis of the research literature published between 2012 and 2020.

Keywords: Literature review; Early mathematics learning, Early mathematics teaching; Technology; Early childhood teachers.

1. Introduction

In the past few years there has been an internationally growing interest in early childhood mathematics education (ECME). The interest in this field is induced firstly by the strong emphasis given on early childhood education in many countries. This is evident by the increase of their expenses and investments on early childhood education and of their access to pre-primary education (Kagan & Roth, 2017). The well documented, positive relation between children's early mathematical knowledge and their later success in mathematics learning is another factor for the growing research in this field (Dunkan et al., 2007).

This survey has been designed to establish an in-depth and comprehensive review of the state-of-the-art of the most important developments and contributions between 2012 and 2020, and of new perspectives and emerging challenges in ECME.

In the survey we identified six major research themes in recent literature on ECME. Three of these themes are content-oriented: number sense and whole number

---

[1] Department of Education, University of Cyprus, Nicosia, Cyprus. E-mail: elia.iliada@ucy.ac.cy
[2] Department of Mathematics, University of Pisa, Pisa, 56127, Italy. E-mail: anna.baccaglinifrank@unipi.it
[3] Department of Mathematics, Science, and Technology Education, Tel Aviv University, Tel Aviv, Israel. E-mail: levenso@tauex.tau.ac.il
[4] Faculty of Education, Chiba University, Chiba, Japan. E-mail: matsuo@faculty.chiba-u.jp
[5] Research and Postgraduate Studies, University of Venda, Thohoyandou, South Africa. E-mail: nosisi.piyose@gmail.com

development, geometry education and children's competences in other content domains. Another theme that is systematically reviewed deals with the role of technologies in early mathematics teaching and learning. A cognition-oriented theme focuses on cognitive skills associated with mathematics learning and special education and comprises two parts: Abilities predictive of or associated with mathematical performance and special education. Finally, the sixth theme focuses on developments and trends in teacher-related issues, particularly on early childhood teachers' knowledge, education and affective issues in mathematics.

To identify relevant research for the survey, we drew from a broad range of sources, focusing on publications since 2012. For each identified theme we searched for relevant peer-reviewed papers in journals in two fields, that is, Mathematics Education and Early Childhood Education, for relevant chapters in prominent research books on Mathematics Education (such as PME Handbook, POEM, ICME-13 monographs), international peer-reviewed conference proceedings, including ICME, PME and CERME. Moreover, we used publications found in search engines using strings with relevant keywords for each theme.

After eliminating double records, we produced annotated bibliography with summaries of the papers that have been identified as relevant for each theme, leading to a comprehensive analysis of the issues raised by this research literature and to a qualitative synthesis of the pertinent findings. This led to the production of a written overview of the research in each theme. A summary of the key findings for each theme of the review and a final section with our concluding remarks are presented below.

2. Number Sense and Whole Number Development

Number sense development is globally recognized as the fundamental foundational knowledge for children's mathematical growth. Hence, literature argues for children's stimulation of numerosity as early as their toddler stage for future benefits. The complexities brought forward by diverse backgrounds of children as well as diverse provisions of stimulation enriches strategies and seek more conceptualization. Children's diverse numerical abilities reflect children's varied experiences from home and their immediate environment (Ramani & Siegler, 2011). These abilities are foundational blocks for children's development of numerical fluency, on the other hand low performance is proven to be associated with limited numerical experiences prior to kindergarten and inability to catch up with peers (Aunio et al., 2015).

Innate abilities of young children have been identified to be observable as early as six months. New-borns of 7 to 94 hours demonstrated that they could map space, number and time including brightness, and loudness (de Hevis et al., 2014). Robertson, Shi and Melancon (2012) discovered that 24 months babies were able to match objects with the defining number. Also, Norwegian toddlers demonstrated competencies using number words, however reciting lower competencies than previous literature (Reikerås, Løge, & Knivsberg, 2012).

Sella et al. (2017) explored numerosity and spatial mapping to three groups, preschool children, 4-year-olds, 1st Grade and 3rd Grade, to discover that spatial mapping favored high numerical abilities in all groups studied. Benz (2014) revealed that 4- to 6-year-olds were able to explain structures in quantities and why they used to compose or decompose, while a more recent study with the use of eye tracking by Schöner and Benz (2018) showed that children built structures in the collection of objects but could not explain their approaches and resort to counting as a strategy.

Strategies that are revealed to enhance numerical abilities are linear board games (Ramani & Siegler, 2011), numerical acuity and inhibitory control, tablet-based non-symbolic approximate arithmetic game, conceptual subitising (Sayers et al., 2016), fine motor skills (Asakawa et al., 2017), story problems (Jordan et al., 2012), catch up numeracy (Holmes & Dowker, 2013) differentiated approach in using games (Bay-Williams & Kling, 2014). Children who get exposed to numeracy stimulation before school demonstrate a positive gain in early schooling (Clerking & Gilligan, 2018; Segers et al., 2015). Particularly, benefits are observed in play-based approach for numeracy development at home, allowing children to learn through play and give opportunity to adults to pose challenging questions and listen to children illustrating their action and adding meaning to them (Magnusson & Pramling, 2018). Furthermore, mediation of patterns and structure influences children numerical fluency positively especially those who are low performers (Lüken & Kampmann, 2018). Problem solving was found to boost four basic operations with an effect size of 0.60 (Bicknell et al., 2016).

Dynamic assessment is one of the strong predictors for problem solving of word problems development (Seethaler et al., 2012). Polotskaia and Savard (2018) used Relational Paradigm in facilitating problem solving and this led to improved problem-solving skills and enabled students to solve problems demanding rational thinking. A longitudinal study favored students who begin to use derived fact strategies during mid-year than those using counting strategies (Gaidoschik, 2012). White and Szucs (2012) promoted modelling methods to increase understanding and developing mental representation through estimation tactics.

Although children's mathematical development through technology is systematically analyzed in a distinct theme of this review, it is worthwhile here to present a number of technology-based interventions that were found to support the numeracy abilities of children with diverse backgrounds. A Math Shelf intervention using a tablet was designed for at risk 4-year-old pre-schoolers with results that show significant improvement performance (Schacter et al., 2016). Also, an adaptive computer game "Number Race" yielded similar findings as Math Shelf intervention for disadvantaged children (Sella et al., 2016). Lady Bug Count and Fingu apps allow children to develop own mathematical concepts such as subitizing, estimation and finger motor-skills (Ladeland Kortenkamp, 2014).

Generally, this literature addresses vital concepts within number sense that need attention and recognition. Our findings indicate a need to understand or unpack children's language as some studies assert that children were able to articulate their strategies and reasons behind their selection, while others report that children were limited to counting in describing their strategies. Furthermore, there is too little literature on transitioning from informal numerosity to formal numerosity and how mediation should be structured to achieve the transitioning.

3. Geometry Education

The literature review about geometry education in early childhood focused on the following major threads: Spatial skills and their relation to mathematics and geometry learning, shape knowledge and understanding, embodied, dynamic and semiotic approaches in geometrical thinking and learning and enhancing and assessing geometry learning.

Regarding the first thread of the review, strong relations have been found between spatial skills and quantitative competences, including, number-line estimation, counting and arithmetic (addition and subtraction) in children already from age 3 up to 7 years (e.g., Verdine et al., 2017). The potential significance of young children's spatial training or learning in their mathematics performance is also highlighted (e.g., Hawes et al., 2015). The relationships between spatial skills and geometry learning are scarcely investigated and research on this issue suggests that the associations between geometry learning and teaching and spatial reasoning are quite complex (e.g., Dindyal, 2015). However, spatial knowledge is found to be the basis for building geometrical knowledge and understandings in problem solving (Soury-Lavergne & Maschietto, 2015).

Our findings on shape knowledge and understanding (second thread) suggest that a great deal of studies focused on plane shapes, taking a rather static perspective, e.g., studying children’s shape recognition and sorting abilities, definitions/descriptions of shapes (e.g., Olkunet al., 2017). A major finding, revealed also by previous literature, is that children develop visual prototypes and they use these prototypes to compare shapes they are asked to identify or to draw (e.g., Dağlı et al. 2014).

A few studies adopted a more spatial perspective in investigating children’s competences in geometry. Research on children’s competences with 3D shapes has shown that children encounter difficulties when reasoning about plane and solid shapes across various kinds of geometric representations (Hallowell et al., 2015) and also in reconstructing a 3D figure with building blocks and in using the regularities of this geometrical object when needed (Maj-Tatsis & Swoboda, 2017).

A large part of research on embodied, dynamic and semiotic approaches in geometrical thinking and learning (third thread) investigated the role of body and gestures in children’s learning of geometry. A major finding is that children’s body and gestures reflect implicit knowledge and have a crucial and fundamental role in the development of geometrical reasoning, in solving geometrical problems and argumentation and in communicating geometrical/spatial relationships (e.g., Calero et al., 2019; Elia, 2018). The multimodality of children’s learning, and specifically the synergy between talk, gesture, and material environment is regarded as a critical characteristic of children's development in geometry by many research studies (e.g., Thom, 2018). A number of studies on dynamic learning environments (DLE) in early geometry provided evidence for the potential of DLE to support children's developing understanding, and reasoning about, different geometry concepts, including the properties of specific shapes, angles and reflective symmetry (e.g., Ng & Sinclair, 2015).

Research on enhancing early geometry learning (fourth thread) suggested play as a significant learning approach. Using children's play as starting point to teach mathematical content supported children's explorations of shapes (Bäckman, 2016), while guided play also enhanced children's shape knowledge (Fisher et al., 2013). In addition, using picture book reading with or without the inclusion of additional mathematical activities was found to be a promising avenue to contribute to the development of children's understanding of shapes and spatial relationships (McGuire et al., 2021). Regarding the assessment of children's geometrical thinking and learning, a study by Thom and McGarvey (2015) showed that drawing serves as a means to access, assess, and attend to children's understanding. Tirosh et al. (2013) highlighted the need to use a combination of tasks to assess strengths and weaknesses of children's geometric knowledge, as not all children take advantage of the opportunities afforded by a given task.

The findings of the review on early geometry education indicate the need for further research on the body's role in children's geometrical learning (regarding both plane and 3D shapes), so as to deepen and enrich current knowledge about how children think and build geometrical understandings, on teaching strategies in geometry and spatial reasoning (e.g., picture books, DLE) to support children move into more abstract ways of thinking and, finally, on how early geometrical knowledge and understanding affect children's future mathematical performance.

4. Children's Competencies in Other Content Domains

Compared to the domains of number and geometry, other content domains have received less attention within ECME research. The major content domains within ECME that are studied in the reviewed literature are patterns and structures, measurement, statistical reasoning and early algebraic reasoning. A common focus of the reviewed studies across the different content domains is twofold: Firstly, offering insights into young children's competences and development and secondly, proposing and investigating the effectiveness of programs or interventions on children's learning. Research on patterns had increased rapidly since 2013, when Mulligan and Mitchelmore (2013) proposed Early Awareness of Mathematical Pattern and Structure showing that it generalizes across early mathematical concepts. In this study, not only can students' abilities of structural development be reliably categorized through particular levels, but students who show a superior level of development on one task also operate at a similar level on other tasks.

Besides this, research has investigated children's recognition of the unit of repeat and the structure of the repeating patterns and the relations with other mathematical contents, such as number and arithmetic, algebra, calculation, or geometrical thinking. It is shown that effective patterning instruction included instruction on symmetrical patterns, patterns with increasing numbers of elements, and patterns involving the rotation of an object through 6 or 8 positions. Moreover, the abilities to ascertain the structure of patterns and understand mathematical language were found as strong predictors of mathematical success, with the latter making a more significant contribution.

Concerning measurement, a significant number of studies in this theme have focused on general measurement, length, area, mass, and time. However, few research papers are about bulk, or volume. Regarding length measurement, which has received greater attention than measurement of other properties, research has provided evidence for the effects of different pedagogical approaches on kindergarten children's learning and evaluated and elaborated the developmental progression or levels of thinking on length measurement (e.g., Szilágyi, etc., 2013). A few pieces of literature are about area measurement. Clements et al. (2018) investigated the effects of instructional interventions through different levels of a learning trajectory designed to support young children's understanding of area measurement as a structuring process.

The literature on data modeling includes research on statistical reasoning, statistical learning, quantitative modeling, and probabilistic thinking. English and Crevensten (2013) explored data modeling with a specific focus on structuring and representing data, including the use of conceptual and meta-representation competence, informal inference, and the role of context through the longitudinal study of data modeling in grades one to three.

Kieran, Pang, Schifter and Ng (2016) investigated the nature of the research carried out in early algebra and how it has shaped the field's growth for the younger students, aged from about six years to 12 years. This study found that mathematical relations, patterns, and arithmetical structures lie at the heart of early algebraic activity, with noticing, conjecturing, generalizing, representing, justifying, and communicating central to students' engagement.

Based on the strengths that have been identified in this current research body about mathematics learning in other content domains in early childhood contexts, we consider important for future research pathways to investigate the connections of the development of early knowledge and skills in content domains beyond number, e.g., pattern and structure, data modeling and early algebra with future mathematical performance of students. Further research could be carried out about the nature of classroom culture and the role of the teacher, the curriculum, instructional strategies and new technologies on the learning in the above mathematical content domains, as well as on the relationship between children's competences in these content domains and their cognitive skills and affective characteristics.

5. The Role of Technology in Mathematics Teaching and Learning

A large body of research on this theme focused on design features of different technological tools used in ECME, such as spreadsheets, Interactive Whiteboards (IWBs), dynamic geometry software and programmable toy robots, but the vast majority of the technological tools described are interactive applets for tablets.

An issue raised in many papers is the need for designing new apps for learning mathematics in order to create constructive opportunities to represent mathematical objects so that they can be manipulated. A second issue raised is the importance of well-founded guidelines at the basis of this design.

A number of studies focused on multi-touch devices, in which environments are designed supporting embodied and multi-player interactions. Among the most referenced and studied is the app TouchCounts, which takes advantage of the multi-touch and gestures functionalities of the device and provides multimodal visual and auditory feedback for every touch or gesture (e.g., De Freitas & Sinclair, 2017). Other studies have focused on activities with programmable toy robots designed to ameliorate preschool children's visuospatial reasoning, engaging them in a playful and tangible way (e.g., Di Lieto et al., 2017).

Frameworks used in this domain or developed from design research include the Theory of Semiotic Mediation (e.g., Bartolini Bussi & Baccaglini-Frank, 2015), the discursive approach, and a framework that makes a distinction between instructive, manipulable, and constructive multimedia (Goodwin & Highfield, 2013), through which a review of educational apps was conducted. An important conclusion was the value of open-ended tasks, also expressed in other studies.

An important area of research is that of computer assisted interventions. Various studies focused on educational software centered around specific topics, such as number sense. This research aimed at highlighting the educational potential (in terms of students' improvements in mathematical achievement) offered by certain apps that exploit affordances of multi-touch devices for fostering preschoolers' development of number-sense (e.g., Baccaglini et al., 2020).

Studies suggested that well-planned integration of apps in the classroom, with clear learning objectives and appropriate feedback could motivate children, enhance concentration, and support independent learning and communication (e.g., De Freitas & Sinclair, 2017; Kaur & Sinclair, 2014). Some studies showed that children make sense of the digital tools and are able to apply the tools purposefully as long as they also interact with an adult, within their zone of proximal development. Findings also suggest that when technology integration is accomplished successfully in early childhood education settings, children tend to interact more with one another and exchange information related to computer tasks as well as to the overall classroom on-going curriculum themes. Research on using dynamic geometry environments with young children suggested that gestures and motion play an important role in children's developing the mathematical conceptions at stake (e.g., Kaur & Sinclair, 2014).

Many of the studies in this theme explored the roles that preschool teachers give to technologies in mathematics education and the ways in which they structure their mathematics learning activities when using technological artefacts. As far as the teachers are concerned, having a negative attitude towards technological artefacts like an IWB led to a decrease in the likelihood to enrich the learning environment and lead to pedagogical change. The IWB does not seem to pose pedagogical challenges to teachers as its stable location offers the opportunity of using it in traditional teaching ways. Instead, tablets seem to pose a problem for some teachers because of their mobility and the need to reconfigure the organization and, to some extent, the roles of teacher and students.

Many semiotic resources were used with different mediating roles in different teaching-learning processes. However, an emerging challenge is that teachers frequently do not seem to be confident about their ability to teach mathematics using computers. These studies suggested providing effective professional learning and development programs so that teachers can employ a wider range of pedagogical strategies to support the children's use of ICT (e.g., Dong, 2018).

Overall, the findings on preschoolers' interactions with touch-screen-based virtual manipulative mathematics apps confirm the hypothesis that multi-touch technology has the potential to foster important aspects of children's development of number-sense, however such research is still in its infancy. A few studies have pointed to relationships between children's strategies used when interacting with certain apps that afford multi-touch inputs and their development of number sense abilities (e.g., Holgersson et al., 2016).

6. Cognitive Skills and Special Education of Young Children

6.1. Abilities predictive of or associated with mathematical performance

In this area of research, a distinction is made between domain general abilities (e.g., executive functions (EF), working memory (WM), long term memory, visuo-spatial abilities, inhibition), domain specific abilities (neurocognitive trend) and abilities related to the socio-cultural dimension and language.

Studies on domain general abilities highlighted the correlations between children's executive functions and their numerical abilities. Also, non-verbal number sense and working memory are central for early mathematical achievement in preschool. Both visuo-spatial working memory and the phonological loop have been associated to mathematical achievement as well as to the children's development of language, and linguistic competence, is also correlated with mathematical achievement (e.g., Fuchs et al., 2019)

Low visuo-spatial abilities do not seem to change the nature of the mental number line, but they can lead to a decrease in its accuracy. On the other hand, visualizing spatial arrangements is a key ability in the development of number sense.

Regarding domain specific abilities, studies have found that the nature and time needed for the development of children's mapping of the new symbolic representations of numbers they learn onto pre-existing non-symbolic representations are not yet clear. Moreover, there seems to be a tendency within neurocognitive research to support the development of cardinality before ordinality. However, some studies have suggested the importance of early focus also on ordinality (e.g., Coles & Sinclair, 2017).

The approximate number system (ANS) plays a key role in the development of basic numerical abilities, and it is thought to help children achieve cardinality. However, an important role is also played by children's spontaneous focus on numerosity (SFON) (e.g., Verschaffel et al., 2016). Research has widely recognized the importance of using fingers, suggesting that explicit teaching of finger counting in early primary school practices should be promoted to help weaker children overcome their difficulties in arithmetic.

The literature on the abilities related to the socio-cultural dimension and language indicated that the role of language is quite controversial. All of the studies analyzed suggested that there are links between early mathematical achievement and phonological competence. Moreover, the use of narration enhances mathematical learning.

Difficulties with mathematical language can also become obstacles in the development of mathematical meanings. This line of research highlighted the key role played by the teacher. Research converged on the main objective of developing multimodal approaches to mathematics education, within which language is a resource that involves various elements (signs, gestures, etc.) combining mathematical symbols (including formal ones) and images to promote the development of mathematical meanings.

Different ways of pronouncing numbers in different languages can support or hinder the development of number sense itself; for example, using languages with more transparent ways of denominating numbers has a long-lasting benefit on children and it places them at an advantage (e.g., Dunbar et al., 2017).

A number of studies emphasized how a deprived socio-economical-cultural environment has a significant negative influence on the development of number sense.

6.2. Special education

The reviewed research in special education within ECME involves two major directions: The first direction is focused on mathematical capabilities and their development in young students with special needs and the second one refers on ways to support and improve mathematics learning of young students with special needs. For both directions, the findings from the literature review reveal a greater emphasis on researching low-attaining children than high-achieving children. Considering the first direction, a large part of research in low performing students focused on students with mathematical learning disabilities (MLD) mainly with respect to numeracy. Several studies have investigated the relations between MLD and cognitive skills (e.g., working memory), language skills, other learning difficulties (e.g., dyslexia, non-verbal learning disabilities) or pathological disabilities. Regarding the second direction, a significant number of studies developed remedial numeracy programs or examined the effectiveness of interventions for young children at risk for mathematics difficulties or low-attaining children in mathematics (e.g., van Garderen et al., 2020). However, the remediation of MLD in educational contexts is a field that needs further improvement. There is also growing research that takes place on the use and effects of evidence-based instructional tools, ICT, teacher professional development, family-centered practices and parent involvement/ training on the mathematical development of children at risk for mathematics difficulties which could contribute to this endeavor.

7. Early Childhood Teachers' Knowledge, Education and Affective Issues in Mathematics

For preschool teachers, subject matter knowledge (SMK) often refers to the concepts and skills learned before first grade, such as number and operation, measurement, geometry, data representations, and patterns. Several studies presented models and frameworks for investigating teachers' knowledge and competencies for teaching early childhood mathematics (Gasteiger & Benz, 2018; Tsamir, et al., 2014). In addition to content, studies included elements of pedagogical content knowledge (PCK) such as knowledge of early mathematics development, ability to observe mathematical situations and to take appropriate pedagogical actions. Lindmeier et al. (2016) added reflective competence and action-related competence. Most studies of preschool teachers' SMK reported on general scores, (e.g., Opperman et al., 2016), with a few focusing on specific domains, such as patterns (e.g.,Tirosh et al., 2017) and geometry (e.g., Tsamir et al., 2015; Ulusoy, 2020). Investigation of teachers' content knowledge in areas such as measurement and data representation is scarce.

Studies that concern teachers' PCK are more frequent than studies of teachers' content knowledge, focusing on teachers' knowledge of students as mathematics learners (Tanase & Wang, 2013) and teachers' knowledge of task orchestration (Hundeland et al., 2017). Several studies focused on teachers' abilities to recognize mathematical situations that occur during children's natural play (e.g., Benz, 2016) or during play-based scenarios (Lee, 2017). In general, teachers note concepts related to classification, number sense, and measurement.

Teachers' attitudes towards teaching mathematics improved through professional development (e.g., Sumpter, 2020). Some teachers claim that mathematics in kindergarten is important because children need to be prepared for first grade (Schuler et al., 2013). Teachers mostly consider counting, rather than geometry, as relevant and important (Schuler et al., 2013).

Concerning pedagogical mathematical beliefs, teachers stress children's need to use their bodies as tools for learning mathematics, for example by climbing up and down to feel differences in height (Franzén, 2014). Most teachers believe that it is essential for children to be active learners (Li et al., 2019) and that the teachers' roles are to ask questions (Cross Francis, 2015), instill curiosity (Schuler et al., 2013), and encourage children to think and draw conclusions by themselves (Li et al., 2019).

Self-beliefs, such as anxiety, confidence, and self-efficacy were found to be linked to various contextual factors (e.g., Thiel & Jenssen, 2018). Gasteiger and Benz (2018) pointed out that teachers' attitudes and motivation influence how they use their knowledge and skills. Higher knowledge is associated with higher levels of student-centered beliefs (Ren & Smith, 2018). Tirosh et al. (2017) investigated the links between pattern knowledge and corresponding self-efficacy.

Few studies investigated the preparation of prospective teachers to teach mathematics in preschool. Instead, programs aimed at promoting practicing teachers' knowledge of their young students' mathematical thinking, and helping teachers develop and use tools to better understand children's engagement with mathematics. Several programs used video as a tool for reflection (e.g., Cross Francis, 2015). Other programs encouraged teachers to use clinical interviews (e.g., Polly et al., 2018).

In conclusion, although this section is about teachers, the common thread running through most studies is an emphasis on children. More studies focused on teachers' PCK than their SMK, and within PCK, research has focused on coming to know young students as mathematics learners. Studies of teachers' beliefs investigated the relevance of mathematics for young children, and what is appropriate mathematics for children. Finally, many professional development programs focused on enhancing teachers' knowledge of children's mathematical reasoning. Further research might focus on teachers' knowledge and beliefs related to the use of technology in preschool mathematics and interventions specifically for prospective preschool teachers.

8. Concluding Remarks

Our work on this survey has shown that there is a plethora of research work of a broad scope on ECME and that there will be continued growth and important progress in this field in the years to come. In the past few years we gained considerable amount of relevant knowledge about what mathematics children know and (can) learn before or at the beginning of formal education, how they learn mathematics and develop their mathematics skills, how early mathematics learning can be stimulated and enhanced and also on teachers' knowledge, acts and beliefs related to early years mathematics. We expect to see further development in the aforementioned areas with greater research attention on the following aspects: assessing and developing children's competences in content strands beyond number sense and also children's cognitive abilities that are strongly associated with mathematics performance (e.g., SFON, ANS), toddlers' mathematical development and learning opportunities, the use of digital tools to support children's learning and opportunities offered to children to engage in embodied ways of mathematical thinking and learning. The development of early childhood teachers' knowledge and skills on the above aspects would be another major challenge in the field.

Acknowledgments

We wish to thank Giulia Lisarelli and Alessandro Ramploud for their help in reviewing a part of the literature for the survey.

References

A. Asakawa, T. Murakami, and S. Sugimura (2017). Effect of fine motor skills training on arithmetical ability in children. *European Journal of Developmental Psychology*, 16(3), 290–301.

P. Aunio, P. Heiskari, J. E. Van Luit, and J. M. Vuorio (2015). The development of early numeracy skills in kindergarten in low-, average-and high-performance groups. *Journal of Early Childhood Research*, 13(1), 3–16.

A. Baccaglini-Frank, G. Carotenuto, and N. Sinclair (2020). Eliciting preschoolers' number abilities using open, multi-touch environments. *ZDM Mathematics Education*, 52(4), 779–791.

K. Bäckman (2016). Children's Play as a Starting Point for Teaching Shapes and Patterns in the Preschool. In: T. Meaney, O. Helenius, M. Johansson, T. Lange and A. Wernberg (eds.), *Mathematics Education in the Early Years*, pp. 223–234. Cham: Springer.

M. G. Bartolini Bussi and K. Baccaglini-Frank (2015). Geometry in early years: sowing seeds for a mathematical definition of squares and rectangles. *ZDM Mathematics Education*, 47(3), 391–405.

J. M. Bay-Williams and G. Kling (2014). Enriching Addition and Subtraction Fact Mastery Through Gam. *Teaching Children Mathematics*, 21(4), 238–247.

C. Benz (2014). Identifying quantities — Children's Constructions to Compose Collections from Parts or Decompose Collections into Parts. In: U. Kortenkamp, B. Brandt, C. Benz, G. Krummheuer, S. Ladel and R. Vogel (eds.), *Early Mathematics Learning*, pp. 189–203. New York, NY: Springer.

C. Benz (2016). Reflection: An Opportunity to Address Different Aspects of Professional Competencies in Mathematics Education. In: T. Meaney, O. Helenius, M. L. Johansson, T. Lange, and A. Wernberg (eds.), *Mathematics Education in the Early Years: Results from the POEM2 Conference*, 2014, pp. 419–435. Springer.

B. Bicknell, J. Young-Loveridge, and N. Nguyen (2016). A design study to develop young children's understanding of multiplication and division. *Mathematics Education Research Journal*, 28(4), 567–583.

C. I. Calero, D. E. Shalom, E. S. Spelke, and M. Sigman (2019). Language, gesture, and judgment: Children's paths to abstract geometry. *Journal of Experimental Child Psychology*, 177, 70–85.

D. H. Clements, J. Sarama, D. W. Van Dine, J. E. Barrett, C. J. Cullen, A. Hudyma, R. Dolgin, A. L. Cullen, and C. L. Eames (2018). Evaluation of three interventions teaching area measurement as spatial structuring to young children. *The Journal of Mathematical Behavior*, 50, 23–41.

A. Clerkin and K. Gilligan (2018). Preschool numeracy plays as a predictor of children's attitudes towards mathematics at age 10. *Journal of Early Childhood Research*, 16(3), 319–334.

A. Coles and N. Sinclair (2017). Re-Thinking 'Normal' Development in the Early Learning of Number. *Journal of Numerical Cognition*, 4(1), 136–158.

D. I. Cross Francis (2015). Dispelling the notion of inconsistencies in teachers' mathematics beliefs and practices: A 3-year case study. *Journal of Mathematics Teacher Education*, 18(2), 173–201.

Ü. Y. Dağlı and E. Halat (2016). Young Children's Conceptual Understanding of Triangle. *Eurasia Journal of Mathematics, Science & Technology Education*, 12(2), 189–202.

E. De Freitas and N. Sinclair (2017). Mathematical gestures: Multitouch technology and the indexical trace. In: T. Dooley and G. Gueudet (eds.), *Proceedings of the 10th CERME*, pp. 2531–2538. Dublin: Institute of Education, Dublin City University, Ireland, and ERME.

M. D. de Hevis, V. Izard, A. Coubart, E. S. Spelke, and A. Streri (April, 2014). Representations of space time and number in neonates. *Proceeding of the National Academy of Sciences of the United States of America*, 111(13), 4809–4813. National Academy of Sciences.

M. C. Di Lieto, et al. (2017). Educational Robotics intervention on Executive Functions in preschool children: A pilot study. *Computers in Human Behavior*, 71, 16–23.

J. Dindyal (2015). Geometry in the early years: a commentary. *ZDM*, 47(3), 519–529.

C. Dong (2018). Preschool teachers' perceptions and pedagogical practices: young children's use of ICT. *Early Child Development and Care*, 188(6), 635–650.

K. Dunbar, A. Ridha, O. Cankaya, C. Jiménez Lira, and J-A. LeFevre (2017). Learning to count: structured practice with spatial cues supports the development of counting sequence knowledge in 3-year-old English-speaking children. *Early Education and Development*, 28(3), 308–322.

G. J. Duncan, C. J. Dowsett, A. Claessens, K. Magnuson, A. C. Huston, P. Klebanov, ... and C. Japel (2007). School readiness and later achievement. *Developmental Psychology*, 43(6), 1428–1446.

I. Elia (2018). Observing the Use of Gestures in Young Children's Geometric Thinking. In: I. Elia, J. Mulligan, A. Anderson, A. Baccaglini-Frank and C. Benz (eds.), *Contemporary Research and Perspectives on Early Childhood Mathematics Education, ICME-13 Monographs*, pp. 159–182. Cham: Springer.

L. D. English and N. Crevensten (2013). Reconceptualizing Statistical Learning in the Early Years. In: L. D. English and J. T. Mulligan (eds.), *Reconceptualizing Early Mathematics Learning*, pp. 67–82. Dordrecht: Springer.

K. R.Fisher, K. Hirsh‐Pasek, N. Newcombe, and R. M. Golinkoff (2013). Taking shape: Supporting preschoolers' acquisition of geometric knowledge through guided play. *Child development*, 84(6), 1872–1878.

K. Franzén (2014). Under-threes' mathematical learning — teachers' perspectives. *Early Years*, 34(3), 241–254.

L. Fuchs, D. Fuchs, P. M. Seethaler, and M. A. Barnes (2019). Addressing the role of working memory in mathematical word-problem solving when designing intervention for struggling learners. *ZDM Mathematics Education*, 52(1), 87–96.

M. Gaidoschik (2012). First-Graders' Development of Calculation Strategies: How Deriving Facts Helps Automatize Facts. *Journal of Math Didakt*, 33(2), 287–315.

H. Gasteiger and C. Benz (2018). Enhancing and analyzing kindergarten teachers' professional knowledge for early mathematics education. *Journal of Mathematical Behavior*, 51, 109–117.

K. Goodwin and K Highfield, K. (2013). A framework for examining technologies and early mathematics learning. In: L. English (ed.) et al., Reconceptualizing early mathematics learning. *Advances in Mathematics Education*, pp. 205–226. Dordrecht: Springer.

D. A. Hallowell, Y. Okamoto, L. F. Romo, and J. R. La Joy (2015). First-graders' spatial-mathematical reasoning about plane and solid shapes and their representations. *ZDM*, 47(3), 363–375.

Z. Hawes, J. Moss, B. Caswell, B. and D. Poliszczuk (2015). Effects of mental rotation training on children's spatial and mathematics performance: A randomized controlled study. *Trends in Neuroscience and Education*, 4(3), 60–68.

I. Holgersson, et al (2016). Fingu — A Game to Support Children's Development of Arithmetic Competence: Theory, Design and Empirical Research. In: P. S. Moyer-Packenham (ed.), *International Perspectives on Teaching and Learning Mathematics with Virtual Manipulatives*, pp. 123–146. Mathematics Education in the Digital Era. Springer.

P. S. Hundeland, I. Erfjord, and M. Carlsen (2017). A kindergarten teacher's revealed knowledge in orchestration of mathematical activities. In: T. Dooley and G. Gueudet (eds.), *Proceedings of the Tenth Congress of the European Society for Research in Mathematics Education*, pp. 1853–1860. Institute of Education, Dublin City University and ERME.

N. Jordan, J. Glutting, N. Dyson, B. Hassinger-Das, and C. Irwin (2012). Building Kindergarteners' number sense: A randomized controlled study. *Journal of Educational Psychology*, 104(3), 647–660.

S. L. Kagan and J. L. Roth (2017). Transforming early childhood systems for future generations: Obligations and opportunities. *International Journal of Early Childhood*, 49, 137–154.

H. Kaur and N. Sinclair (2014). Young children's thinking about various types of triangles in a dynamic geometry environment. In: P. Liljedahl, C. Nicol, S. Oesterle and D. Allan (eds.), *Proceedings of the Joint Meeting of PME 38 and PME-NA 36*. Vol. 3, pp. 409–416. Vancouver, Canada: PME.

C. Kieran, J. S. Pang, D. Schifter, and S. F. Ng (2016). *Early Algebra: Research into its Nature, its Learning and its Teaching*. Dordrecht: Springer.

S. Ladel and U. Kortenkamp (2014). Number Concepts — Processes of Internalization and Externalization by the Use of Multi-Touch Technology. In: U. Kortenkamp, B Brandt, C. Benz, G. Krummheuer, S. Ladel and R. Vogel (eds.), *Early Mathematics Learning*, pp. 237–253. New York, NY: Springer.

J. E. Lee (2017). Preschool teachers' pedagogical content knowledge in mathematics. *International Journal of Early Childhood*, 49(2), 229–243.

X. Li, K. McFadden, and M. DeBey (2019). Is it DAP? American preschool teachers' views on the developmental appropriateness of a preschool math lesson from China. *Early Education and Development*, 30(6), 765–787.

A. Lindmeier, B. Hepberger, A. Heinze, and E. Moser Opitz (2016). Modeling cognitive dispostions of educators for early mathematics education. In: C. Csíkos, A. Rausch and J. Szitányi (eds.), *Proceedings of the 40th Conference of the International Group for the Psychology of Mathematics Education*. Vol. 3, pp. 219–226. PME.

M. M. Lüken and R. Kampmann (2018). The Influence of Fostering Children's Patterning Abilities on Their Arithmetic Skills in Grade 1. In: I. Elia, J. Mulligan, A. Anderson, A. Baccaglini-Frank and C. Benz (eds.), *Contemporary Research and Perspectives on Early Childhood Mathematics Education, ICME-13 Monographs*, pp. 55–66. Cham: Springer.

B. Maj-Tatsis and E. Swoboda (2017). Discovering regularities in a geometrical objects environment. In: T. Dooley and G. Gueudet (eds.), *Proceedings of the Tenth Congress of the European Society for Research in Mathematics Education*, pp. 1869–1876. Dublin, Ireland: Institute of Education, Dublin City University and ERME.

M. Magnusson and N. Pramling (2018). In 'Numberland': play based pedagogy in response to imaginative numeracy. *International Journal of Early Years Education*, 26(1), 24–41.

A. Maier and C. Benz (2014). Children's Constructions in the Domain of Geometric Competencies (in Two Different Instructional Settings). In: U. Kortenkamp, B. Brandt, C. Benz, G. Krummheuer, S. Ladel and R. Vogel (eds.), *Early Mathematics Learning*, pp. 173–187. New York, NY: Springer.

P. McGuire, B. Himot, G. Clayton, M. Yoo, and M. E. Logue (2021). Booked on Math: Developing Math Concepts in Pre-K Classrooms Using Interactive Read-Alouds. *Early Childhood Education Journal*, 49(2), 313–323.

J. T. Mulligan and M. C. Mitchelmore (2013). Early Awareness of Mathematical Pattern and Structure. In: L. D. English and J. T. Mulligan (eds.), *Reconceptualizing Early Mathematics Learning*, pp. 29–47. Dordrecht: Springer.

O. Ng and N. Sinclair (2015). Young children reasoning about symmetry in a dynamic geometry environment. *ZDM Mathematics Education*, 47(3), 421–434.

S. Olkun, E. Swoboda, P. Vighi, Y. Yuan, and B. Wollring (2017). Topic Study Group No. 12: Teaching and Learning of Geometry (Primary Level). In: G. Kaiser (ed.), *Proceedings of the 13th International Congress on Mathematical Education*, pp. 429–433. Cham: Springer.

E. Oppermann, Y. Anders, and A. Hachfeld (2016). The influence of preschool teachers' content knowledge and mathematical ability beliefs on their sensitivity to mathematics in children's play. *Teaching and Teacher Education*, 58, 174–184.

D. Polly, C. Wang, C. Martin, R. Lambert, D. Pugalee, and C. Middleton (2018). The influence of mathematics professional development, school-level, and teacher-level variables on primary students' mathematics achievement. *Early Childhood Education Journal*, 46(1), 31–45.

E. Polotskaia and A. Savard (2018). Using the Relational Paradigm: effects on pupils' reasoning in solving additive word problems. *Research in Mathematics Education*, 20(1), 70–90.

G. B. Ramani and R. S. Siegler (2011). Reducing the gap in numerical knowledge between low-and middle-income preschoolers. *Journal of applied developmental Psychology*, 32(3), 146–159.

E. Reikerås, I. K. Løge, and A. M. Knivsberg (2012). The mathematical competencies of toddlers expressed in their play and daily life activities in Norwegian kindergartens. *International Journal of Early Childhood*, 44(1), 91–114.

L. Ren and W. M. Smith (2018). Teacher characteristics and contextual factors: links to early primary teachers' mathematical beliefs. *Journal of Mathematics Teacher Education*, 21(4), 321–350.

E. K. Robertson, R. Shi, and A. Melançon (2012). Toddlers use the number feature in determiners during online noun comprehension. *Child development*, 83(6), 2007–2018.

J. Sayers, P. Andrews, and L. Björklund Boistrup (2016). The Role of Conceptual Subitising in the Development of Foundational Number Sense. In: T. Meaney, O. Helenius, M. Johansson, T. *Lange and A. Wernberg (eds.), Mathematics Education in the Early Years*, pp. 371–394. Cham: Springer.

J. Schacter, J. Shih, C. M. Allen, L. DeVaul, A. B. Adkins, T. Ito, and B. Jo (2016). Math shelf: A randomized trial of a prekindergarten tablet number sense curriculum. *Early Education and Development*, 27(1), 74–88.

P. Schöner and C. Benz (2018). Visual Structuring Processes of Children When Determining the Cardinality of Sets: The Contribution of Eye-Tracking. In: C. Benz, A. Steinweg, H. Gasteiger, P. Schöner, H. Vollmuth and J. Zöllner (eds.), *Mathematics Education in the Early Years*, pp. 123–143. Cham: Springer.

S. Schuler, N. Kramer, R. Kröger, and G. Wittmann, G. (2013). Beliefs of Kindergarten and primary school teachers towards mathematics teaching and learning. In: B. Ubuz, Ç. Haser, and M. Alessandra Mariotti (eds.), *Proceedings of the Eighth Congress of*

European Research in Mathematics Education (CERME 8), pp. 2128–2137. Middle East Technical University.

P. M. Seethaler, L. S. Fuchs, D. Fuchs, and D. L. Compton (2012). Predicting first graders' development of calculation versus word-problem performance: The role of dynamic assessment. *Journal of educational psychology*, 104(1), 224.

E. Segers, T. Kleemans, and L. Verhoeven (2015). Role of Parent Literacy and Numeracy Expectations and Activities in Predicting Early Numeracy Skills. *Mathematical Thinking and Learning*, 17(2–3), 219–236.

F. Sella, I. Berteletti, D. Lucangeli, and M. Zorzi (2017). Preschool children use space, rather than counting, to infer the numerical magnitude of digits: Evidence for a spatial mapping principle. *Cognition*, 158, 56–67.

F. Sella, P. Tressoldi, D. Lucangeli, and M. Zorzi (2016). Training numerical skills with the adaptive videogame "The Number Race": A randomized controlled trial on preschoolers. *Trends in Neuroscience and Education*, 5(1), 20–29.

S. Soury-Lavergne and M. Maschietto (2015). Articulation of spatial and geometrical knowledge in problem solving with technology at primary school. *ZDM*, 47(3), 435–449.

S. T. Stacy, M. Cartwright, Z. Arwood, J. P. Canfield, and H. Kloos (2017). Addressing the math-practice gap in elementary school: are tablets a feasible tool for informal math practice? *Frontiers in Psychology*, 8, 179.

Sumpter, L. (2020). Preschool educators' emotional directions towards mathematics. International Journal of Science and Mathematics Education, 18(6), 1169–11984.

J. Szilágyi, D. H. Clements, and J. Sarama (2013). Young Children's Understandings of Length Measurement: Evaluating a Learning Trajectory. *Journal for Research in Mathematics Education*, 44(3), 581–620.

M. Tanase and J. Wang (2013). Knowing students as mathematics learners and teaching numbers 10-100: A case study of four 1st grade teachers from Romania. *The Journal of Mathematical Behavior*, 32(3), 564–576.

O. Thiel and L. Jenssen (2018). Affective-motivational aspects of early childhood teacher students' knowledge about mathematics. *European Early Childhood Education Research Journal*, 26(4), 512–534.

J. S. Thom (2018). (Re)(con)figuring Space: Three Children's Geometric Reasonings. In: I. Elia, J. Mulligan, A. Anderson, A. Baccaglini-Frank and C. Benz (eds.), *Contemporary Research and Perspectives on Early Childhood Mathematics Education, ICME-13 Monographs*, pp. 131–158. Cham: Springer.

J. S. Thom and L. M. McGarvey (2015). The act and artifact of drawing (s): observing geometric thinking with, in, and through children's drawings. *ZDM*, 47(3), 465–481.

D. Tirosh, P. Tsamir, E. Levenson, R. Barkai, and M. Tabach (2017). Defining, drawing, and continuing repeating patterns: Preschool teachers' self-efficacy and knowledge. In: C. Andra, D. Brunetto, E. Levenson and P. Liljedahl (eds.), *Teaching and Learning in Maths Classrooms — Emerging themes in affect-related research: Teachers' beliefs, students' engagement and social interaction*, pp. 17–26. Springer International Publishing.

D. Tirosh, P. Tsamir, E. Levenson, M. Tabach, and R. Barkai (2013). Two children, three tasks, one set of figures: highlighting different elements of children's geometric knowledge. In: B. Ubuz, Ç. Haser and M. Alessandra Mariotti (eds.), *Proceedings of the Eighth Congress of the European Society for Research in Mathematics Education*, pp. 2238–2237. Ankara, Turkey: Middle East Technical University.

P. Tsamir, D. Tirosh, E. Levenson, R. Barkai, and M. Tabach (2015). Early-years teachers' concept images and concept definitions: triangles, circles, and cylinders. *ZDM Mathematics Education*, 47(3), 497–509.

P. Tsamir, D. Tirosh, E. Levenson, M. Tabach, and R. Barkai (2014). Employing the CAMTE Framework: Focusing on preschool teachers' Knowledge and Self-efficacy Related to Students' Conceptions. In: U. Kortenkamp, B. Brandt, C. Benz, G. Krummheuer, S. Ladel and R. Vogel (eds), *Early Mathematics Learning: Selected Papers of the POEM 2012 Conference*, pp. 291–306. Springer.

F. Ulusoy (2020). Prospective Early Childhood and Elementary School Mathematics Teachers' Concept Images and Concept Definitions of Triangles. *International Journal of Science and Mathematics Education*, 19(5), 1057–1078.

D. van Garderen, J. K. Lannin, and J. Kamuru (2020). Intertwining special education and mathematics education perspectives to design an intervention to improve student understanding of symbolic numerical magnitude. *Journal of Mathematical Behavior*, 59, 100782.

B. N. Verdine, R. M. Golinkoff, K. Hirsh-Pasek, and N. S. Newcombe (2017). Spatial skills, their development, and their links to mathematics. *Monographs of The Society for Research in Child Development*, 82(1), 7–30.

L. Verschaffel, E. Lehtinen, and W. Van Dooren (2016). Neuroscientific studies of mathematical thinking and learning: a critical look from a mathematics education viewpoint. *ZDM Mathematics Education*, 48(3), 385–391.

S. L. J. White and D. Szucs (2012). Representational change and strategy use in children's number line estimation during the first years of primary school. *Behavioral and Brain Functions*, 8(1), 1–12.